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DETERMINATION OF LOSS FACTOR FOR BEAMS WITH VISCOELASTIC LAYER BY MEANS OF METHODS BASED ON WAVE PROPAGATION

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Abstract. *Problem of estimation of loss factor for a beam covered with viscoelastic layer is addressed in the paper. Two estimation methods based on an analysis of a wave field generated in a vibrating structure are tested. Wave fields postulated theoretically in the methods are fit to the one reconstructed from responses measured along length of the beam. Complex wave number is changed in consecutive iterations until satisfactory agreement is reached. Optimum value of complex wave number is used to calculate loss factor from theoretically derived expressions. Reliability and limitations of the methods are investigated. Estimation error is analysed. Obtained values of loss factor are compared with results of standard Oberst test.*

1 INTRODUCTION

Problem of estimation of a damping parameter for beams with viscoelastic layer is addressed in the paper. H. Oberst, who as a first attacked the problem, considered thin metal sheets covered with a layer of adhesive viscoelastic material [1,2]. He derived relation between loss factor of a composite and component layers. Analogical expressions but for a plate covered with viscoelastic layer placed under elastic constraining layer were given by Kerwin, Ross and Ungar[3,4]. Oberst developed also experimental procedure for measurement of composite loss factor. Later works refer to the method as half-value bandwidth. It still serves as a basic tool for estimation of loss factor but it suffers all inherent disadvantages of modal formulation. Due to drastic increase of modal density in higher frequencies, observed in all structures, the approach is limited to low band covering first few modes of vibrations. Moreover, only values of the damping parameter, corresponding to natural frequencies, can be estimated by means of the test.

The paper focuses on another class of methods, called k-space, based on analysis of wavefield generated in a vibrating structure. The name underlines the fact of introduction of complex wave number as a way to include damping. Inhomogenous Wave Correlation method[5,6] and solution proposed by McDaniel et al[7,8] are used independently to determine frequency variation of both complex wave number and loss factor. The methods fit theoretically predicted wavefield with the one reconstructed from measurements done in spatially distributed points. Experimental data come from a test conducted with the use of cantilever beam covered with viscoelastic layer and excited by random continuous signal. In IWC technique the complex wave number is found by a search of space of allowed values. Best guess is used as a starting value in optimisation algorithm employed in McDaniel method. Influence of measurement noise on obtained results is taken into account by calculation of coherence function and its introduction into estimation procedure. As theoretical description of wavefield proposed in IWC method lacks terms corresponding to nearfield, it is investigated how neglecting of measurements done in the vicinity of boundaries affect performance of the method. For comparative reasons values of loss factor determined in Oberst test are also demonstrated.

Section 2 and 3 show theoretical bases of IWC and McDaniel method, respectively. Experimental procedure is described in section 4. The paper concludes with discussion of results and final remarks.

2 THEORETICAL FORMULATION OF IWC METHOD

IWC method was originally developed as a tool for identification of direction-dependent dispersion curves in plane structures. As a one-directional propagation is assumed, description of a wavefield simplifies significantly and takes form:

$$W(x) = e^{i\bar{k}x} \tag{1}$$

for a given frequency ω, where $\bar{k}$ is a complex wave number. Relation between wave number and loss factor η is given by ratio of phase and group velocity, denoted by c_φ and c_g, respectively[9]:

$$\frac{\mathrm{Im}\{\bar{k}\}}{\mathrm{Re}\{\bar{k}\}} = \eta \frac{c_\varphi}{2c_g}\,. \tag{2}$$

Since for bending waves group velocity is twice the phase velocity, loss factor may be expressed directly in terms of wave number:

$$\eta = 4\frac{\mathrm{Im}\{\bar{k}\}}{\mathrm{Re}\{\bar{k}\}} \ . \tag{3}$$

Correlation between wavefield observed in experiment and the one described theoretically is calculated for every frequency by means of the formula:

$$IWC(\bar{k}) = \frac{\left|\sum_{i=1}^{n} W^*(x_i,\bar{k}) \cdot \tilde{W}(x_i)\rho(x_i)\right|}{\sqrt{\sum_{i=1}^{n}\left|W(x_i,\bar{k})\right|^2 \sum_{i=1}^{n}\left|\tilde{W}(x_i)\right|^2 \rho(x_i)}} \ , \tag{4}$$

where $\tilde{W}(x_i)$, $\rho(x_i)$ stand for transfer function and coherence, respectively, both calculated for response at x_i, * denotes complex conjugate and x_i is a location of ith measurement point. Value of $\bar{k}$ is determined by maximisation of ratio (4). Finite set of pairs $(\mathrm{Re}\{\bar{k}\},\mathrm{Im}\{\bar{k}\})$ is searched for solution, no optimisation algorithm is used.

3 THEORETICAL FORMULATION OF MCDANIEL METHOD

The method is based on analysis of vibrations of Bernoulli beam. Homogenous governing equation is used:

$$EJ\,\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \ . \tag{5}$$

After being Fourier transformed, the equation (5) becomes:

$$E\big(1 + i\eta(\omega)\big)J\,\frac{\partial^4 W(x,\omega)}{\partial x^4} - \omega^2 \rho A W(x,\omega) = 0 \ , \tag{6}$$

where introduced term $E\big(1 - i\eta(\omega)\big)$ describes damping. Solution

$$W(x,\omega) = c_1(\omega)e^{i\bar{k}x} + c_2(\omega)e^{-i\bar{k}x} + c_3(\omega)e^{\bar{k}x} + c_4(\omega)e^{-\bar{k}x} \tag{7}$$

includes complex wave number:

$$\bar{k} = \sqrt[4]{\frac{\rho A}{E\big(1 + i\eta(\omega)\big)J}\,\omega^2} \ . \tag{8}$$

If the theoretical model is correct, relation (7) should be satisfied for every measured response.

Thus, a set of equations written in a matrix form for a given frequency ω

$$\begin{bmatrix} W(x_1) \\ .. \\ W(x_i) \\ .. \\ W(x_n) \end{bmatrix} = \begin{bmatrix} e^{i\bar{k}x_1} & e^{-i\bar{k}x_1} & e^{kx_1} & e^{-kx_1} \\ .. & .. & .. & .. \\ e^{i\bar{k}x_i} & e^{-i\bar{k}x_i} & e^{kx_i} & e^{-kx_i} \\ .. & .. & .. & .. \\ e^{i\bar{k}x_n} & e^{-i\bar{k}x_n} & e^{kx_n} & e^{-kx_n} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{9}$$

ought hold, where x_i , $i = 1,..,n$ denotes a location of ith measurement point and $W(x_i, \omega)$ is a response obtained at x_i. Unknown constants c_j, $j = 1,..,4$ are determined by means of least square method for each complex value of $\bar{k}$ assumed in consecutive loops of optimisation procedure. Original objective function, proposed by McDaniel et al, has been modified. Now it contains coherence function ρ:

$$\varepsilon(\bar{k}) = \frac{\sqrt{\sum_{i=1}^{n} \left| W(x_i, \bar{k}) - \tilde{W}(x_i) \right|^2 \rho(x_i)}}{\sqrt{\sum_{i=1}^{n} \left| \tilde{W}(x_i) \right|^2 \rho(x_i)}} , \tag{10}$$

where $\tilde{W}(x_i)$ denotes experimentally obtained response. Nelder-Mead simplex algorithm[10] has been employed to speed up search of minimum. Loss factor may be calculated from expression (8):

$$\eta = \frac{\mathrm{Im}\{\bar{k}^4\}}{\mathrm{Re}\{\bar{k}^4\}} , \tag{11}$$

if $\bar{k}$, minimizing (10), is used.

4 EXPERIMENTAL VALIDATION

Steel sample of dimensions 0.27m×0.02m×0.001m covered with viscoelastic material manufactured by RIETER France was tested. Clamping was realized by means of massive vice. Gearing&Watson V4 shaker with Bruel&Kjaer 8200 force charge mounted on its head, excited the sample at the free end. Polytec OFV 350 laser vibrometer measured velocity in 26 points spaced 0.01m apart along length of the beam. Bruel&Kjaer Pulse multi-analyzer system acquired data. Signal coming from the force charge is used as a reference one in FFT analysis.

Frequency dependence of real and imaginary part of wave number obtained for both presented methods is shown in figures 1 and 2, respectively. 40Hz÷6400Hz band is plotted. The upper frequency is a compromise beween limitations of FFT analysis and a requirement of not exceeding Nyquist frequency. The last condition has to be satisfied if measurement points are evenly spaced, which is the case. However, as it has been demonstrated by

McDaniel et al[8], violation of the requirement is possible for irregular arrangement of the points. Very high level of noise, recorded below 40Hz, reflected in low values of coherence, leads to erroneous results in the range, namely real part of wave number becomes negative and loss factor takes values lying outside an interval $\langle 0,1 \rangle$. Envelope of coherence calculated for all measurement points is shown in figure 3.

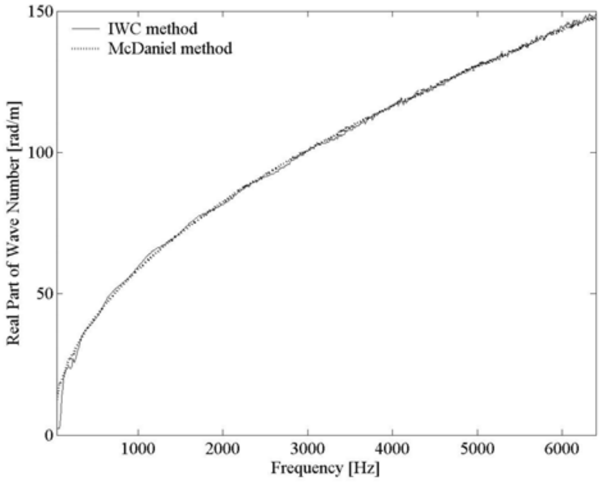

Figure 1. Real part of wave number vs frequency

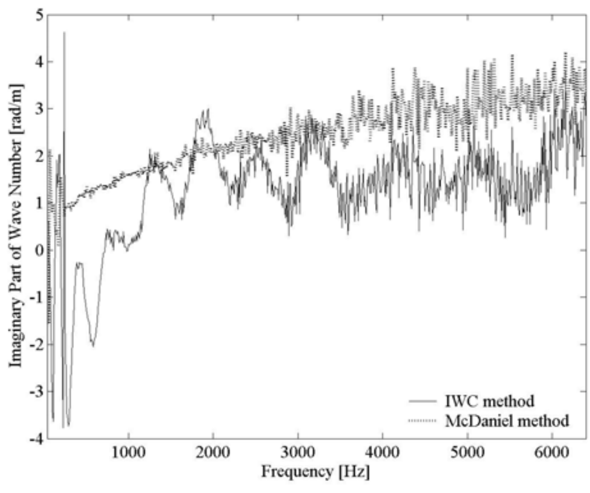

Figure 2. Imaginary part of wave number vs frequency

Shape of the curves plotted in figure 1, representing real part of wave number, agrees very well with the one predicted theoretically for either conservative system or system with hysteretic model of damping applied, within the whole frequency range. Both methods produce almost identical results. However, evident differences, particularly in lower frequencies, are observed in imaginary part of wave number, as it is shown in figure 2.

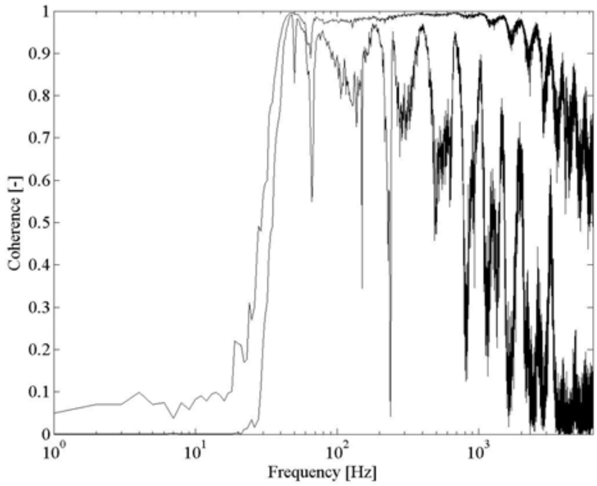

Figure 3. Envelope of coherence

Curves representing complex wave number determined by means of IWC method, have been obtained by use of measurement data acquired in 20 points with exclusion of 6 others, lying closest to the ends of the sample. The incomplete set of data has been used on account of the fact that theoretical solution assumed in the method well describes wavefield existing away from boundaries. Numerical tests conducted for one-dimensional wavefield[5] show that IWC method returns better estimates of imaginary part of wave number if responses measured in the vicinity of boundaries and excitation are discarded. However, this effect is observed only in the case when modal solution of equation (5) is used to generate the wavefield and it disappears if a wave solution is employed. No relevant differences in values of real part of wave number obtained for all mentioned cases are reported.

Results of the numerical tests have been verified experimentally. Curves depicting frequency variation of real and imaginary part of wave number determined by use of IWC method, already presented in figures 1 and 2, are set together in common graph marked as figure 4 and overlapped by respective curves obtained for complete set of measurements.

In order to eliminate influence of measurement noise on obtained values of the parameters, coherence has been included in objective functions of search algorithms used in both methods. Figures 5 and 6 show differences in returned values of real and imaginary part of wave number between the case when coherence is taken into account and the case when it is neglected. Absolute differences, expressed in percents of values of the respective parameters obtained with the use of coherence, are plotted. Logarithmic scale is used for ordinate axis. Gaps in curves depicting differences in real and imaginary part of wave number returned by IWC method correspond to frequencies at which no difference exists. It results in zero value, having no finite representation in logarithmic scale. From figures 5 and 6 it is clearly seen that unlike IWC method, McDaniel method is almost insensitive to measurement noise. It should be underlined that differences in imaginary part of wave number are on average two orders of magnitude bigger than those observed for real part.

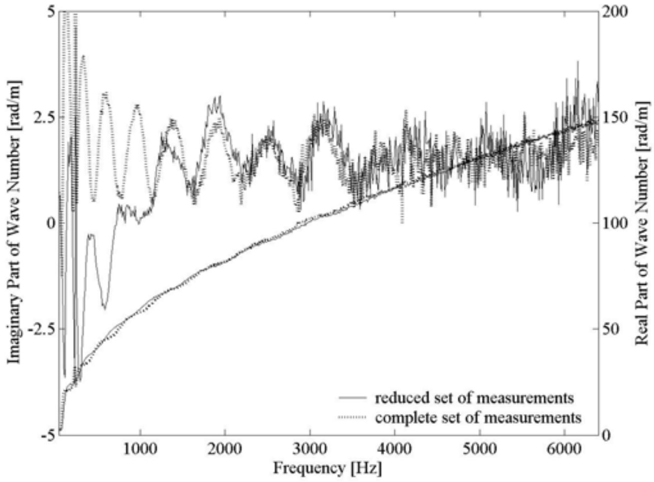

Figure 4. Real and imaginary part of wave number obtained by means of IWC method with the use of all measurements and reduced number of measurements done away from boundaries and excitation

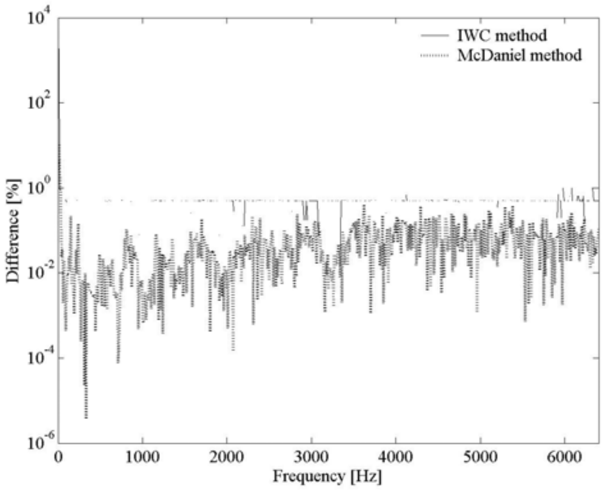

Figure 5. Difference between obtained values of real part of wave number for the cases when coherence is included in objective function and neglected

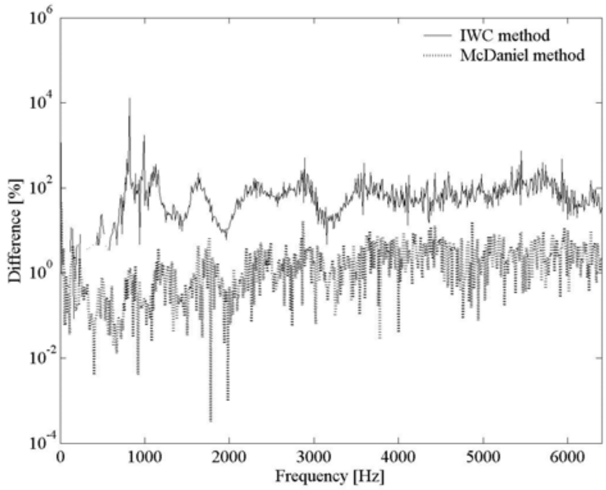

Figure 6. Difference between obtained values of imaginary part of wave number for the cases when coherence is included in objective function and neglected

Loss factor calculated with the aid of formulas (3) and (11) for IWC and McDaniel method, respectively, is plotted in figure 7. The graph covers frequencies from 225Hz to 6400Hz. It is based on data presented in figures 1 and 2. The lower limit of showed frequency range corresponds to second resonant frequency of the tested sample. It has been discovered that precision of McDaniel method drastically decreases below that frequency in the presented case. Exceptionally high variation of imaginary part of wave number, observed in lower frequencies, confronted with high coherence in the range and low value of objective function, displayed in figure 8, indicating good fit of postulated solution to experimental response, proves that the failure of the method is due to its inherent limitations. More profound theoretical analysis of the problem has been already given by McDaniel et al[8].

Performance of IWC method needs some farther comment. Objective function employed in the method takes unit value if theoretically predicted wavefield fits ideally to the real one. The function tends to zero as the fit fails. Since IWC method searches through discretized set of admissible values of the parameters, probability of not finding global optimum is minimised provided that the discretisation is dense enough. From the last remarks and from the fact that very low values of the objective function are obtained, as is shown in figure 8, it is inferred that the description of a wavefield proposed in IWC method does not correspond to the one observed in tests.

Fact that average value of loss factor estimated by means of McDaniel method does not vary with frequency indicates that hysteretic model of damping applies. Results obtained with the use of McDaniel method coincide with values of loss factor determined in standard Oberst test conducted in compliance with DIN 53440, depicted in figure 7 by black dots, what confirms correctness of the parameter returned by the former method.

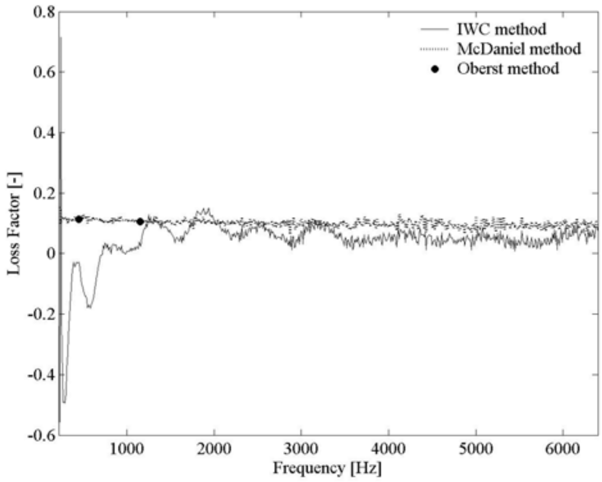

Figure 7. Loss factor vs frequency

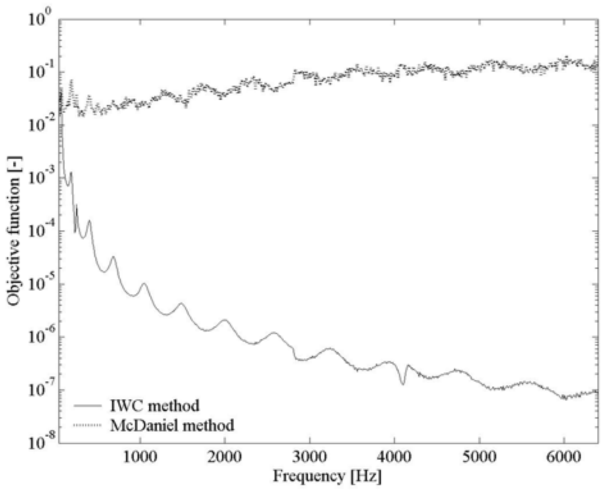

Figure 8. Objective function vs frequency

5 CONCLUSIONS

In the presented paper problem of estimation of damping parameter for the case of cantilever beam covered with viscoelastic layer has been addressed. Two methods based on analysis of a wavefield generated in a system have been investigated. It has been shown that model of a wavefield used in IWC method does not apply to a problem of vibrating beam. Thus values of returned parameters are not reliable. In spite of the fact that it is based on model of homogenous beam, McDaniel method enables correct estimation of loss factor for the composite within wide frequency range. Moreover, thanks to introduction of loss factor notation, it is possible to identify model of damping by means of that method.

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